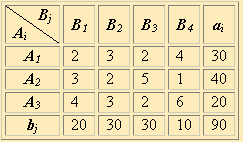
**Лабораторная работа №4.1**

**Оптимальность плана транспортной задачи**

**Задача 1.**



**Метод минимальной стоимости**

| **Bj**  **Ai** | **B1** | | **B2** | | **B3** | | **B4** | | **ai** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A1** |  | 2 |  | 3 |  | 2 |  | 4 | 30 |
| **20** | |  | | **10** | |  | |
| **A2** |  | 3 |  | 2 |  | 5 |  | 1 | 40 |
| **0** | | **30** | |  | | **10** | |
| **A3** |  | 4 |  | 3 |  | 2 |  | 6 | 20 |
|  | |  | | **20** | |  | |
| **bj** | 20 | | 30 | | 30 | | 10 | | 90 |

𝑛 + 𝑚 − 1 = 4 + 3 − 1 = 6 > 5, т.е. получен вырожденный план. Следовательно, заполняем числом «0» пустую клетку 𝐴2𝐵1, т.к. она имеет минимальный тариф (𝑐21 = 3)

Получаем опорный план:

| 20 | 0 | 10 | 0 |
| --- | --- | --- | --- |
| 𝑋 = ( 0 | 30 | 0 | 10) |
| 0 | 0 | 20 | 0 |

𝑍 = 20 ∗ 2 + 10 ∗ 2 + 30 ∗ 2 + 10 ∗ 1 + 20 ∗ 2 = 170

Составим систему уравнений потенциалов:

| 𝑢1 + 𝑣1 = 2 | Полагая 𝑢1 = 0, найдем | 𝑢2 = 1 |
| --- | --- | --- |
| 𝑢1 + 𝑣3 = 2 |  | 𝑢3 = 0 |
| 𝑢2 + 𝑣1 = 3 |  | 𝑣1 = 2 |
| 𝑢2 + 𝑣2 = 2 |  | 𝑣2 = 1 |
| 𝑢2 + 𝑣4 = 1 |  | 𝑣3 = 2 |
| 𝑢3 + 𝑣3 = 2 |  | 𝑣4 = 0 |

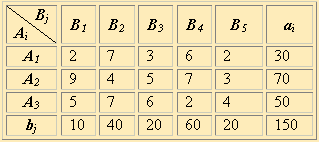
| **Bj**  **Ai** | **B1** | | **B2** | | **B3** | | **B4** | | **ai** | ***u*** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A1** |  | 2 |  | 3 |  | 2 |  | 4 | 30 | **0** |
| **20** | | **2** | | **10** | | **4** | |
| **A2** |  | 3 |  | 2 |  | 5 |  | 1 | 40 | **1** |
| **0** | | **30** | | **2** | | **10** | |
| **A3** |  | 4 |  | 3 |  | 2 |  | 6 | 20 | **0** |
| **2** | | **2** | | **20** | | **6** | |
| **bj** | 20 | | 30 | | 30 | | 10 | | 90 |  |
| ***v*** | **2** | | **1** | | **2** | | **0** | |  |  |



Все разности ∆𝑐𝑖𝑗 ≥ 0, следовательно, **план является оптимальным**

𝑍𝑚𝑖𝑛 = 170

**Задача 2.**



**Метод минимальной стоимости**

| **Bj**  **Ai** | **B1** | | **B2** | | **B3** | | **B4** | | **B5** | | **ai** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A1** |  | 2 |  | 7 |  | 3 |  | 6 |  | 2 | 30 |
| **10** | |  | | **0** | |  | | **20** | |
| **A2** |  | 9 |  | 4 |  | 5 |  | 7 |  | 3 | 70 |
|  | | **40** | | **20** | | **10** | |  | |
| **A3** |  | 5 |  | 7 |  | 6 |  | 2 |  | 4 | 50 |
|  | |  | |  | | **50** | |  | |
| **bj** | 10 | | 40 | | 20 | | 60 | | 20 | | 150 |

𝑛 + 𝑚 − 1 = 5 + 3 − 1 = 7 > 6, т.е. получен вырожденный план. Следовательно,

заполняем числом «0» пустую клетку 𝐴1𝐵3, т.к. она имеет минимальный тариф (𝑐13 = 3)

Получаем опорный план:

| 10 | 0 | 0 | 0 | 20 |
| --- | --- | --- | --- | --- |
| 𝑋 = ( 0 | 40 | 20 | 10 | 0 ) |
| 0 | 0 | 0 | 50 | 0 |

𝑍 = 10 ∗ 2 + 40 ∗ 4 + 20 ∗ 5 + 10 ∗ 7 + 50 ∗ 2 + 20 ∗ 2 = 490

Составим систему уравнений потенциалов:

| 𝑢1 + 𝑣1 = 2  𝑢1 + 𝑣3 = 3  𝑢1 + 𝑣5 = 2 | Полагая 𝑢1 = 0, найдем | 𝑢2 = 2  𝑢3 = −3  𝑣1 = 2 |
| --- | --- | --- |
| 𝑢2 + 𝑣2 = 4  𝑢2 + 𝑣3 = 5  𝑢2 + 𝑣4 = 7  𝑢3 + 𝑣4 = 2 |  | 𝑣2 = 2  𝑣3 = 3  𝑣4 = 5  𝑣5 = 2 |

| **Bj**  **Ai** | **B1** | | **B2** | | **B3** | | **B4** | | **B5** | | **ai** | ***u*** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A1** |  | 2 |  | 7 |  | 3 |  | 6 |  | 2 | 30 | **0** |
| **10** | | **5** | | **0** | **+** | **1** | | **20** | − |
| **A2** |  | 9 |  | 4 |  | 5 |  | 7 |  | 3 | 70 | **2** |
| **5** | | **40** | | **20** | − | **10** | | **-1** | **+** |
| **A3** |  | 5 |  | 7 |  | 6 |  | 2 |  | 4 | 50 | **-3** |
| **6** | | **8** | | **6** | | **50** | | **5** | |
| **bj** | 10 | | 40 | | 20 | | 60 | | 20 | | 150 |  |
| ***v*** | **2** | | **2** | | **3** | | **5** | | **2** | |  |  |

∆𝑐𝑖𝑗 = ∆𝑐25 = −1 < 0, следовательно **план не является оптимальным**

∆𝑥 = min(20; 20) = 20

Получим новый опорный план:

| **Bj**  **Ai** | **B1** | | **B2** | | **B3** | | **B4** | | **B5** | | **ai** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A1** |  | 2 |  | 7 |  | 3 |  | 6 |  | 2 | 30 |
| **10** | |  | | **20** | |  | | **0** | |

| **A2** |  | 9 |  | 4 |  | 5 |  | 7 |  | 3 | 70 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | | **40** | |  | | **10** | | **20** | |
| **A3** |  | 5 |  | 7 |  | 6 |  | 2 |  | 4 | 50 |
|  | |  | |  | | **50** | |  | |
| **bj** | 10 | | 40 | | 20 | | 60 | | 20 | | 150 |

𝑛 + 𝑚 − 1 = 5 + 3 − 1 = 7 > 6, т.е. получен вырожденный план. Следовательно, заполняем числом «0» пустую клетку 𝐴1𝐵5, т.к. она имеет минимальный тариф (𝑐15 = 2)

Составим систему уравнений потенциалов:

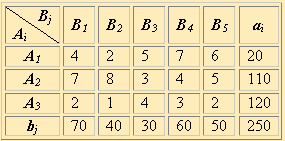
| 𝑢1 + 𝑣1 = 2 | Полагая 𝑢1 = 0, найдем | 𝑢2 = 1 |
| --- | --- | --- |
| 𝑢1 + 𝑣3 = 3 |  | 𝑢3 = −4 |
| 𝑢1 + 𝑣5 = 2 |  | 𝑣1 = 2 |
| 𝑢2 + 𝑣2 = 4 |  | 𝑣2 = 3 |
| 𝑢2 + 𝑣4 = 7 |  | 𝑣3 = 3 |
| 𝑢2 + 𝑣5 = 3 |  | 𝑣4 = 6 |
| 𝑢3 + 𝑣4 = 2 |  | 𝑣5 = 2 |

| **Bj**  **Ai** | **B1** | | **B2** | | **B3** | | **B4** | | **B5** | | **ai** | ***u*** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A1** |  | 2 |  | 7 |  | 3 |  | 6 |  | 2 | 30 | **0** |
| **10** | | **4** | | **20** | | **0** | | **0** | |
| **A2** |  | 9 |  | 4 |  | 5 |  | 7 |  | 3 | 70 | **1** |
| **6** | | **40** | | **1** | | **10** | | **20** | |
| **A3** |  | 5 |  | 7 |  | 6 |  | 2 |  | 4 | 50 | **-4** |
| **7** | | **8** | | **7** | | **50** | | **6** | |
| **bj** | 10 | | 40 | | 20 | | 60 | | 20 | | 150 |  |
| ***v*** | **2** | | **3** | | **3** | | **6** | | **2** | |  |  |

Все разности ∆𝑐𝑖𝑗 ≥ 0, следовательно, **план является оптимальным**

𝑍𝑚𝑖𝑛 = 10 ∗ 2 + 20 ∗ 3 + 0 ∗ 2 + 40 ∗ 4 + 10 ∗ 7 + 20 ∗ 3 + 50 ∗ 2 = 470

**Задача 3.**



**Метод минимальной стоимости**

| **Bj**  **Ai** | **B1** | | **B2** | | **B3** | | **B4** | | **B5** | | **ai** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A1** |  | 4 |  | 2 |  | 5 |  | 7 |  | 6 | 20 |
|  | |  | |  | |  | | **20** | |
| **A2** |  | 7 |  | 8 |  | 3 |  | 4 |  | 5 | 110 |
|  | |  | | **30** | | **60** | | **20** | |
| **A3** |  | 2 |  | 1 |  | 4 |  | 3 |  | 2 | 120 |
| **70** | | **40** | |  | |  | | **10** | |
| **bj** | 70 | | 40 | | 30 | | 60 | | 50 | | 250 |

𝑛 + 𝑚 − 1 = 5 + 3 − 1 = 7, что совпадает с количеством занятых клеток т.е. получен невырожденный план.

𝑍 = 70 ∗ 2 + 40 ∗ 1 + 30 ∗ 3 + 60 ∗ 4 + 20 ∗ 6 + 20 ∗ 5 + 10 ∗ 2 = 750

Составим систему уравнений потенциалов:

| 𝑢1 + 𝑣5 = 6  𝑢2 + 𝑣3 = 3  𝑢2 + 𝑣4 = 4  𝑢2 + 𝑣5 = 5  𝑢3 + 𝑣1 = 2 | Полагая 𝑢1 = 0, найдем | 𝑢2 = −1  𝑢3 = −4  𝑣1 = 6  𝑣2 = 5  𝑣3 = 4 |
| --- | --- | --- |
| 𝑢3 + 𝑣2 = 1  𝑢3 + 𝑣5 = 2 |  | 𝑣4 = 5  𝑣5 = 6 |

| **Bj**  **Ai** | **B1** | | **B2** | | **B3** | | **B4** | | **B5** | | **ai** | ***u*** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A1** |  | 4 | **-3** | 2 |  | 5 |  | 7 |  | 6 | 20 | **0** |
| **-2** | | **+** | **1** | | **2** | | **20** | − |
| **A2** |  | 7 | **4** | 8 |  | 3 |  | 4 |  | 5 | 110 | **-1** |
| **2** | |  | **30** | | **60** | | **20** | |

| **A3** |  | 2 |  | 1 |  | 4 |  | 3 |  | 2 | 120 | **-4** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **70** | | **40** | − | **4** | | **2** | | **10** | **+** |
| **bj** | 70 | | 40 | | 30 | | 60 | | 50 | | 250 |  |
| ***v*** | **6** | | **5** | | **4** | | **5** | | **6** | |  |  |



∆𝑐𝑖𝑗 = ∆𝑐12 = −3 < 0, следовательно **план не является оптимальным**

∆𝑥 = min(20; 40) = 20

Получим новый опорный план:

| **Bj**  **Ai** | **B1** | | **B2** | | **B3** | | **B4** | | **B5** | | **ai** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A1** |  | 4 |  | 2 |  | 5 |  | 7 |  | 6 | 20 |
|  | | **20** | |  | |  | |  | |
| **A2** |  | 7 |  | 8 |  | 3 |  | 4 |  | 5 | 110 |
|  | |  | | **30** | | **60** | | **20** | |
| **A3** |  | 2 |  | 1 |  | 4 |  | 3 |  | 2 | 120 |
| **70** | | **20** | |  | |  | | **30** | |
| **bj** | 70 | | 40 | | 30 | | 60 | | 50 | | 250 |

Составим систему уравнений потенциалов:

| 𝑢1 + 𝑣2 = 2 | Полагая 𝑢1 = 0, найдем | 𝑢2 = 2 |
| --- | --- | --- |
| 𝑢2 + 𝑣3 = 3 |  | 𝑢3 = −1 |
| 𝑢2 + 𝑣4 = 4 |  | 𝑣1 = 3 |
| 𝑢2 + 𝑣5 = 5 |  | 𝑣2 = 2 |
| 𝑢3 + 𝑣1 = 2 |  | 𝑣3 = 1 |
| 𝑢3 + 𝑣2 = 1 |  | 𝑣4 = 2 |
| 𝑢3 + 𝑣5 = 2 |  | 𝑣5 = 3 |

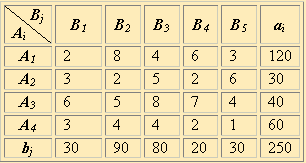
| **Bj**  **Ai** | **B1** | | **B2** | | **B3** | | **B4** | | **B5** | | **ai** | ***u*** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A1** |  | 4 |  | 2 |  | 5 |  | 7 |  | 6 | 20 | **0** |
| **1** | | **20** | | **4** | | **5** | | **3** | |
| **A2** |  | 7 |  | 8 |  | 3 |  | 4 |  | 5 | 110 | **2** |
| **2** | | **4** | | **30** | | **60** | | **20** | |
| **A3** |  | 2 |  | 1 |  | 4 |  | 3 |  | 2 | 120 | **-1** |
| **70** | | **20** | | **4** | | **2** | | **30** | |

| **bj** | 70 | 40 | 30 | 60 | 50 | 250 |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ***v*** | **3** | **2** | **1** | **2** | **3** |  |  |

Все разности ∆𝑐𝑖𝑗 ≥ 0, следовательно, **план является оптимальным**

𝑍𝑚𝑖𝑛 = 20 ∗ 2 + 30 ∗ 3 + 60 ∗ 4 + 20 ∗ 5 + 70 ∗ 2 + 20 ∗ 1 + 30 ∗ 2 = 690

**Задача 4.**



**Метод минимальной стоимости**

| **Bj**  **Ai** | **B1** | | **B2** | | **B3** | | **B4** | | **B5** | | **ai** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A1** |  | 2 |  | 8 |  | 4 |  | 6 |  | 3 | 120 |
| **30** | | **10** | | **80** | |  | |  | |
| **A2** |  | 3 |  | 2 |  | 5 |  | 2 |  | 6 | 30 |
|  | | **30** | |  | |  | |  | |
| **A3** |  | 6 |  | 5 |  | 8 |  | 7 |  | 4 | 40 |
|  | | **40** | |  | |  | |  | |
| **A4** |  | 3 |  | 4 |  | 4 |  | 2 |  | 1 | 60 |
|  | | **10** | |  | | **20** | | **30** | |
| **bj** | 30 | | 90 | | 80 | | 20 | | 30 | | 250 |

𝑛 + 𝑚 − 1 = 5 + 4 − 1 = 8, что совпадает с количеством занятых клеток т.е. получен невырожденный план.

𝑍 = 30 ∗ 2 + 10 ∗ 8 + 30 ∗ 2 + 40 ∗ 5 + 10 ∗ 4 + 80 ∗ 4 + 20 ∗ 2 + 30 ∗ 1 = 830

Составим систему уравнений потенциалов:

| 𝑢1 + 𝑣1 = 2 | Полагая 𝑢1 = 0, найдем | 𝑢2 = −6 |
| --- | --- | --- |
| 𝑢1 + 𝑣2 = 8 |  | 𝑢3 = −3 |
| 𝑢1 + 𝑣3 = 4 |  | 𝑢4 = −4 |
| 𝑢2 + 𝑣2 = 2 |  | 𝑣1 = 2 |
| 𝑢3 + 𝑣2 = 5 |  | 𝑣2 = 8 |
| 𝑢4 + 𝑣2 = 4 |  | 𝑣3 = 4 |

𝑢4 + 𝑣4 = 2 𝑣4 = 6

𝑢4 + 𝑣5 = 1 𝑣5 = 5

| **Bj**  **Ai** | **B1** | | **B2** | | **B3** | | **B4** | | **B5** | | **ai** | ***u*** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A1** |  | 2 | **10** | 8 |  | 4 | **0** | 6 |  | 3 | 120 | **0** |
| **30** | | − | **80** | |  | **-2** | **+** |
| **A2** |  | 3 | **30** | 2 |  | 5 | **2** | 2 |  | 6 | 30 | **-6** |
| **7** | |  | **7** | |  | **7** | |
| **A3** |  | 6 | **40** | 5 |  | 8 | **4** | 7 |  | 4 | 40 | **-3** |
| **7** | |  | **7** | |  | **2** | |
| **A4** |  | 3 | **10** | 4 |  | 4 |  | 2 |  | 1 | 60 | **-4** |
| **5** | | **+** | **4** | | **20** | | **30** | − |
| **bj** | 30 | | 90 | | 80 | | 20 | | 30 | | 250 |  |
| ***v*** | **2** | | **8** | | **4** | | **6** | | **5** | |  |  |

∆𝑐𝑖𝑗 = ∆𝑐15 = −2 < 0, следовательно **план не является оптимальным**

∆𝑥 = min(10; 30) = 10

Получим новый опорный план:

| **Bj**  **Ai** | **B1** | | **B2** | | **B3** | | **B4** | | **B5** | | **ai** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A1** |  | 2 |  | 8 |  | 4 |  | 6 |  | 3 | 120 |
| **30** | |  | | **80** | |  | | **10** | |
| **A2** |  | 3 |  | 2 |  | 5 |  | 2 |  | 6 | 30 |
|  | | **30** | |  | |  | |  | |
| **A3** |  | 6 |  | 5 |  | 8 |  | 7 |  | 4 | 40 |
|  | | **40** | |  | |  | |  | |
| **A4** |  | 3 |  | 4 |  | 4 |  | 2 |  | 1 | 60 |
|  | | **20** | |  | | **20** | | **20** | |
| **bj** | 30 | | 90 | | 80 | | 20 | | 30 | | 250 |

Составим систему уравнений потенциалов:

𝑢1 + 𝑣1 = 2 Полагая 𝑢1 = 0, найдем 𝑢2 = −4

𝑢1 + 𝑣3 = 4 𝑢3 = −1

| 𝑢1 + 𝑣5 = 3 | 𝑢4 = −2 |
| --- | --- |
| 𝑢2 + 𝑣2 = 2 | 𝑣1 = 2 |
| 𝑢3 + 𝑣2 = 5 | 𝑣2 = 6 |
| 𝑢4 + 𝑣2 = 4 | 𝑣3 = 4 |
| 𝑢4 + 𝑣4 = 2 | 𝑣4 = 4 |
| 𝑢4 + 𝑣5 = 1 | 𝑣5 = 3 |



| **Bj**  **Ai** | **B1** | | **B2** | | **B3** | | **B4** | | **B5** | | **ai** | ***u*** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A1** |  | 2 |  | 8 |  | 4 |  | 6 |  | 3 | 120 | **0** |
| **30** | | **2** | | **80** | | **2** | | **10** | |
| **A2** |  | 3 |  | 2 |  | 5 |  | 2 |  | 6 | 30 | **-4** |
| **5** | | **30** | | **5** | | **2** | | **7** | |
| **A3** |  | 6 |  | 5 |  | 8 |  | 7 |  | 4 | 40 | **-1** |
| **5** | | **40** | | **5** | | **4** | | **2** | |
| **A4** |  | 3 |  | 4 |  | 4 |  | 2 |  | 1 | 60 | **-2** |
| **3** | | **20** | | **2** | | **20** | | **20** | |
| **bj** | 30 | | 90 | | 80 | | 20 | | 30 | | 250 |  |
| ***v*** | **2** | | **6** | | **4** | | **4** | | **3** | |  |  |

Все разности ∆𝑐𝑖𝑗 ≥ 0, следовательно, **план является оптимальным**

𝑍𝑚𝑖𝑛 = 30 ∗ 2 + 80 ∗ 4 + 10 ∗ 3 + 30 ∗ 2 + 40 ∗ 5 + 20 ∗ 4 + 20 ∗ 2 + 20 ∗ 1 = 810